Gauss Bessel Beam Mode

Digital Twin Specification

Twin Code:	`SF-GBES01`
Twin Name:	Gauss Bessel Beam Mode
Category:	Source
Type:	Function-Based
Version:	1.0
Package:	Platform
Last Updated:	`2026-02-02`

Description

Generates a Bessel–Gauss beam at its waist plane ($z = 0$). This beam exhibits non-diffracting properties over a user-defined distance (Bessel zone) while remaining physically realizable due to its finite-energy Gaussian envelope. The twin can produce optical vortices with different topological charges ℓ and provides both transverse electric field components $\boldsymbol{E}_x$ and $\boldsymbol{E}_y$ according to a specified Jones vector.

Model Parameters

- **Wavelength** λ: Operating wavelength (e.g., 532 nm).
- **Beam Waist Radius** w_0: $1/e^2$ intensity radius of the Gaussian envelope at the waist plane (e.g., 1 mm).
- **Bessel-Zone Length** $Z_{\max}$: Desired axial distance over which the beam maintains its Bessel-like character (e.g., 10 mm).
- **Ring Number** N (Range: 1 - 20, default: 2): Index of the bright ring used to define $Z_{\max}$. $N = 1$ corresponds to the first bright ring.
- **Topological Charge** ℓ (Range: 0 - 20, default: 0): Integer vortex charge. $\ell = 0$ gives a conventional (non-vortex) Bessel–Gauss beam; $\ell \neq 0$ adds an azimuthal phase $\exp(i\ell\phi)$.
- **Jones Vector**: Polarization state (default: linear horizontal, $[1, 0]^\top$).
- **Sampling Accuracy** (default: 1): Oversampling factor for numerical accuracy. Values >1 increase sampling density.
- **Numerical Window Size Factor** (default: 1): Multiplicative factor applied to the automatically determined simulation window size.

Simulation Model

The twin generates the Bessel–Gauss beam field at its waist plane ($z = 0$) in cylindrical coordinates ($\boldsymbol{\rho}, \phi$):

$$\boldsymbol{E}(\boldsymbol{\rho}, \phi, 0) = A_0\, J_{|\ell|}(k_{\boldsymbol{\rho}}\boldsymbol{\rho})\, \mathrm{e}^{i\ell\phi}\, \exp\left(-\frac{\boldsymbol{\rho}^2}{w_0^2}\right) \mathbf{J}, \tag{1}$$

where:

- $J_{|\ell|}$ is the Bessel function of the first kind of order $|\ell|$,

- $k_{\boldsymbol{\rho}}$ is the transverse wavenumber (radial spatial frequency),
- $\mathbf{J} = [J_x, J_y]^\top$ is the normalized Jones vector defining the polarization,
- A_0 is the field amplitude that scales with the specified beam power.

Bessel Function Extrema Values

The radial positions of intensity maxima (bright rings) are given by $\boldsymbol{\rho}_n = j_{|\ell|,n}/k_{\boldsymbol{\rho}}$, where $j_{|\ell|,n}$ denotes the position of the n-th extremum (local maximum) of $J_{|\ell|}(x)$. Key values are:

Table 1: Bessel function extrema positions $j_{|\ell|,n}$ for common topological charges. For $\ell = 0$, $n = 1$ corresponds to the central peak.

ℓ	Ring n	$j_{|\ell|,n}$ (position)	Description
0	0	0.000	Central maximum at $\boldsymbol{\rho} = 0$
0	1	3.832	First bright ring
0	2	7.016	Second bright ring
0	3	10.173	Third bright ring
1	1	1.841	First ring (dark center)
1	2	5.331	Second bright ring
1	3	8.536	Third bright ring
2	1	3.054	First ring (dark center)
2	2	6.706	Second bright ring
2	3	9.969	Third bright ring

Key Physical Principle: Bessel-Zone Definition

The Bessel-zone length $Z_{\max}$ is defined as the axial distance from the waist where the N-th bright ring (at constant radial position $\boldsymbol{\rho}_N = j_{|\ell|,N}/k_{\boldsymbol{\rho}}$) coincides with the $1/e^2$ radius of the Gaussian envelope:

$$\boldsymbol{\rho}_N = w(Z_{\max}), \tag{2}$$

where $w(z) = w_0\sqrt{1 + (z/z_R)^2}$ is the propagating Gaussian radius with Rayleigh range $z_R = \pi w_0^2/\lambda$.

Substituting $\boldsymbol{\rho}_N = j_{|\ell|,N}/k_{\boldsymbol{\rho}}$ gives:

$$\frac{j_{|\ell|,N}}{k_{\boldsymbol{\rho}}} = w_0\sqrt{1 + \left(\frac{Z_{\max}}{z_R}\right)^2}. \tag{3}$$

Solving for $k_{\boldsymbol{\rho}}$ yields the fundamental design equation:

$$\boxed{k_{\boldsymbol{\rho}} = \frac{j_{|\ell|,N}}{w_0\sqrt{1 + \left(\frac{Z_{\max}}{z_R}\right)^2}}}, \tag{4}$$

which determines the transverse wavenumber from the user inputs (w_0, $Z_{\max}$, N, ℓ).

Parameter Relationships

1. **Core diameter**: The central lobe (for $\ell = 0$) or first bright ring diameter is approximately

$$d_{\text{core}} \approx 2\frac{j^{\text{zero}}_{|\ell|,1}}{k_{\boldsymbol{\rho}}}, \tag{5}$$

where $j^{\text{zero}}_{|\ell|,1}$ is the first zero of $J_{|\ell|}$ (e.g., $j^{\text{zero}}_{0,1} \approx 2.405$).

2. **Rayleigh range vs. Bessel zone**: For $Z_{\max} \ll z_R$, the Gaussian envelope remains nearly constant, yielding a longer effective Bessel zone. For $Z_{\max} \gg z_R$, the envelope expands rapidly, limiting the Bessel zone.

3. **Vortex beams**: When $\ell \neq 0$, the field has a phase singularity at $\boldsymbol{\rho} = 0$ with ℓ intertwined wavefronts.

Typical Application Scenarios

1. **Optical trapping and manipulation**: Bessel–Gauss beams provide extended axial trapping regions compared to Gaussian beams, useful for multi-particle manipulation.

2. **Laser material processing**: The non-diffracting core enables consistent intensity over longer working distances for drilling, cutting, or welding.

3. **Optical coherence tomography (OCT)**: Extended depth of field improves imaging range without refocusing.

4. **Free-space optical communications**: Reduced diffraction spreading maintains beam quality over longer atmospheric paths.

5. **Atomic physics and optical lattices**: Vortex Bessel–Gauss beams ($\ell \neq 0$) transfer orbital angular momentum to atoms.

6. **Microscopy and super-resolution**: Self-reconstructing properties enable imaging through scattering media.

Software Usage

1. **Adding the twin**: From the Digital Twin Hub, search for "Bessel" or "SF-BESS01" and add the Gauss Bessel Beam Mode twin to your VirtualLab Fusion document.

2. **Parameter configuration**: Set λ, w_0, $Z_{\max}$, ℓ, polarization, and beam power according to your application. Use the default $N = 2$ unless specific ring behavior is required.

3. **System construction**: Connect the source twin to component twins (lenses, apertures, etc.) and detector twins (field monitors, power detectors) to build your optical system.

4. **Propagation analysis**: Place a Field Monitor detector at any position behind the source to observe the beam's evolution. Propagation through free space or components is handled automatically by VLF. To investigate axial development, use the Parameter Run feature to sweep the detector position along z, revealing the Bessel-zone behavior and eventual Gaussian divergence.

5. **Focal plane analysis**: To study focusing properties, connect the source to a lens twin and place a detector at the focal plane to examine the Bessel–Gauss beam's Fourier transform.

6. **Sampling adjustment**: The automatic sampling uses the **Sampling Accuracy** and **Numerical Window Size Factor** parameters. Increase these if numerical artifacts (aliasing, interpolation errors) appear. For vortex beams ($\ell \neq 0$), if artifacts appear near the vortex center, increase the **Sampling Accuracy** factor above 1. Source oversampling is automatically compensated during subsequent propagations in VLF.

Author:	LightTrans International GmbH
Contact:	support@lighttrans.com
Keywords:	Bessel, Gaussian, non-diffracting, vortex, structured light, source, orbital angular momentum (OAM), optical trapping, extended depth of field, Bessel zone, sampling, beam power
Related Twins:	CF-BESV01, CF-BESP01, SF-GAUS01, SF-DONA01, SF-DONR01